Estimation of Maximum Liquid Head over Landfill Barriers

Xuede Qian[1]; Donald H. Gray[2]; and Robert M. Koerner[3]

Abstract: To properly design a drainage layer for either landfill leachate collection systems or final covers, the designer must be able to estimate the maximum liquid head over the barrier for any proposed configuration. This paper presents four explicit formulas for estimating the maximum liquid head over an impervious sloping barrier. By means of numeric comparisons, McEnroe's 1993 method is recommended for design of drainage layers for both bottom liners and final covers. Pipe slope is an important parameter that influences the maximum leachate head on the liner. Different combinations of base grade and pipe slope can directly affect the actual drainage behavior. If the pipe slope is steeper than the base grade, it will make a longer drainage distance and cause a high leachate head on the liner. A method for calculating the maximum liquid head in multilayered drainage media (e.g., geosynthetic and soil) is presented in the paper. The key consideration for this case is to determine the equivalent hydraulic conductivity of the combined drainage media under the phreatic surface under unconfined seepage conditions.

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Introduction

The maximum liquid head over a barrier must be estimated at two different locations in the design of a landfill. The first is to calculate the maximum leachate head over the base, or primary liner. This liquid head must not exceed 300 mm based on Federal and state regulations in the United States and as well as in many other countries. The second is to calculate the maximum saturated depth in the final cover system above the barrier layer. This saturated depth is one of the most important parameters influencing the stability of the final cover system.

In general, the hydraulic conductivity of the drainage layer, the drainage slope, and the drainage distance from the upstream boundary to the drainage outlet are relatively easy to determine for a landfill design (Fig. 1). They are constants for a specific landfill. But, the rate of inflow into the landfill leachate drainage layer or final cover drainage layer is variable. Furthermore, the flow conditions in the drainage layers for both the leachate collection system and the final cover system are in an unsteady state. Calculation of liquid head over the barriers for unsteady flow is very complex. In order to simplify calculations and still obtain reliable results, the flow in the drainage layers of the leachate collection system and final cover system are assumed to be at steady state. In that case, the inflow rate will be constant and can be assumed equal to the maximum inflow rate. Thus, the maximum liquid head over the barriers can be calculated based on the maximum inflow rate using a steady-state assumption. If the calculated maximum leachate head over the bottom liner is not greater than 300 mm under these worst case conditions, the leachate head over the liner will always meet the regulatory requirements. Thus, this method will provide conservative results (Qian 1994; Qian et al. 2001).

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Methods for Calculating Maximum Liquid Head

Four methods are currently used to calculate the maximum leachate head over the landfill liner or the maximum saturated depth in the final cover system. Two of these methods were proposed by Moore and the other two by Giroud et al. and McEnroe, respectively. Each will be discussed in a limited amount of detail.

Moore's 1980 Method

An explicit formula for estimating the maximum liquid head over a sloping barrier can be found in several technical guidance documents of the U.S. Environmental Protection Agency (U.S. EPA 1980, 1989). This formula is expressed as follows (Fig. 1):

$$y_{\max}=L\cdot(r/k)^{1/2}[(kS^2/r)+1-(kS/r)(S^2+r/k)^{1/2}] \quad (1)$$

where $y_{\max}$=maximum liquid head on the landfill barrier (mm); L=horizontal drainage distance (mm); r=inflow rate (i.e., rate of vertical inflow to the drainage layer per unit horizontal area) (mm/s); k=hydraulic conductivity of the drainage layer (mm/s); S=slope of the drainage layer ($S=\tan\alpha$); and α=slope angle of drainage layer, measured from horizontal (deg). This formula was first presented by C. A. Moore in 1980 without derivation or explanation of its origin or limitations.

Moore's 1983 Method

Moore presented another formula for estimating the maximum liquid head over a sloping barrier in 1983 (U.S. EPA 1983). This formula is expressed as follows (Fig. 1):

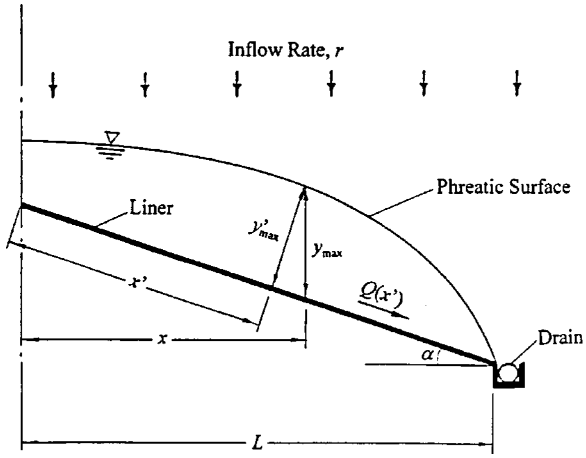

Fig. 1. Phreatic surface in landfill drainage layer

$$y_{\max}=L\cdot[(r/k+S^2)^{1/2}-S] \tag{2}$$

This formula is simpler than Moore's 1980 formula. But, again neither derivation nor explanation of its origin or limitations were included in the 1983 report.

Giroud's 1992 Method

Giroud et al. (1992) used a numerical method to derive a different formula for estimating the maximum liquid head over a sloping liner. This formula is expressed as follows (Fig. 1):

$$y_{\max}=jL[(4r/k+S^2)^{1/2}-S]/(2\cdot\cos\alpha) \tag{3}$$

The parameter j in above formula is called as a numerical modifying factor and defined as follows:

$$j=1-0.12\exp\left\{-\left[\log\left(\frac{1.6r}{kS^2}\right)^{5/8}\right]^2\right\} \tag{4}$$

McEnroe's 1993 Method

Based on the standard Dupuit assumptions, McEnroe presented a graphic method in 1989 (McEnroe 1989) to estimate the maximum leachate head over the liner. It is only suitable for slopes less than 10%. McEnroe presented another set of formulas for estimating the maximum saturated depth over a sloping liner in 1993 (McEnroe 1993). In the derivation of these formulas, the lateral drainage over the liner was described by an extended form of the Dupuit discharge formula (Harr 1962; Childs 1971; Chapman 1980).

The following explicit formulas for estimating the maximum liquid head over a landfill liner that is draining freely with no backwater effect from the collection trough according to McEnroe (1993) are expressed as follows (Fig. 1):

If $R<1/4$ (Fig. 2)

$$y_{\max}=LS\cdot(R-RS+R^2S^2)^{1/2}\cdot\{[(1-A-2R)(1+A-2RS)]/[(1+A-2R)(1-A-2RS)]\}^{1/(2A)} \tag{5}$$

If $R=1/4$ (Fig. 2)

$$y_{\max}=LSR\cdot(1-2RS)/(1-2R)\times\exp\{2R\cdot(S-1)/[(1-2RS)(1-2R)]\} \tag{6}$$

If $R>1/4$ (Fig. 2)

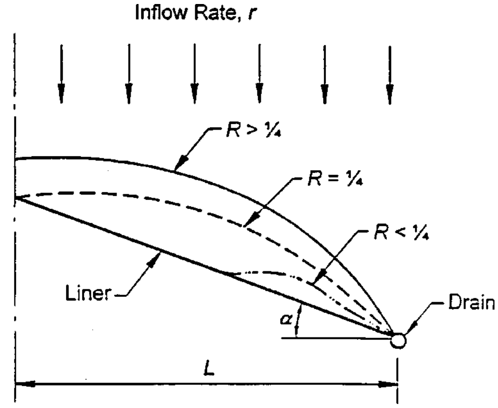

Fig. 2. Phreatic surfaces for different R values

$$y_{\max}=LS\cdot(R-RS+R^2S^2)^{1/2}\exp\{(1/B)\cdot\tan^{-1}[(2RS-1)/B]-(1/B)\cdot\tan^{-1}[(2R-1)/B]\} \tag{7}$$

The parameters R, A, and B used in above formulas are defined as

$$R=r/(k\sin^2\alpha) \tag{8}$$

$$A=(1-4R)^{1/2} \tag{9}$$

$$B=(4R-1)^{1/2} \tag{10}$$

If the drainage system is working properly, i.e., not excessively clogged, the liquid level in the drainage trench will be below the upper edge of the trench, and will have no effect on the saturated-depth profile over the liner. This has been termed the "free drainage condition." Note that all methods described herein are only suitable for the free drainage condition.

Comparisons of Various Calculation Methods

The derivation of McEnroe's 1993 method is based on the extended Dupuit assumptions, whereas Giroud's 1992 method is based on several simplifying assumptions and numerical methods. No derivations and explanations for either Moore's 1980 or 1983 method can be found from the U.S. EPA documents that present these two methods.

Landfill Leachate Drainage Layer

Assume a landfill cell has a hydraulic conductivity of the leachate drainage layer k of 0.01 cm/s, the leachate drainage slope S of 2%, a horizontal drainage distance from the most upstream point to the leachate collection pipe L of 25 m, and an inflow rate r of 3 mm/day. The maximum liquid head over the liner $y_{\max}$ calculated by these different methods are as follows:

Moore's 1980 method

$$y_{\max}=269\ \text{mm}$$

Moore's 1983 method

$$y_{\max}=183\ \text{mm}$$

Giroud's 1992 method

$$y_{\max}=245\ \text{mm}$$

McEnroe's 1993 method

$$y_{\max}=246\ \text{mm}$$

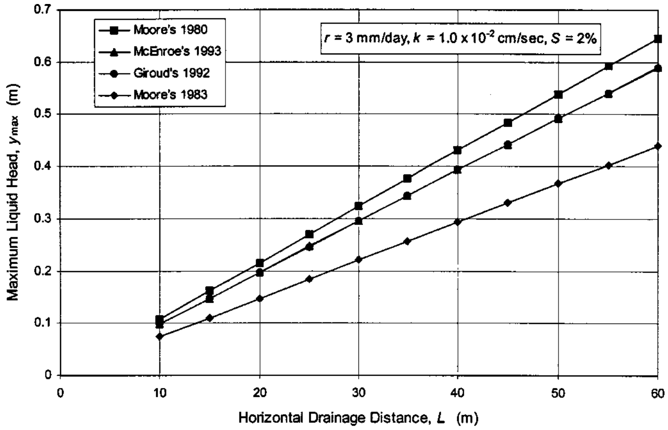

Fig. 3. Relationship between maximum liquid head and horizontal drainage distance for landfill cell from different calculation methods

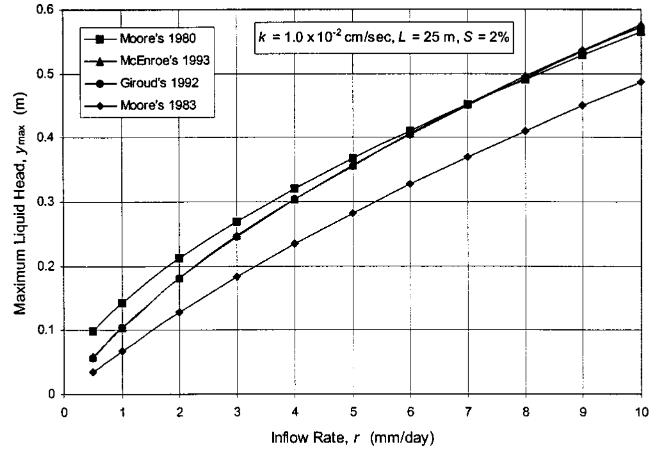

Fig. 5. Relationship between maximum liquid head and inflow rate for landfill cell from different calculation methods

Compared to McEnroe's 1993 method (which is theoretically derived), the above results show that Moore's 1980 method overestimates the maximum leachate head over the liner. In contrast, Moore's 1983 method greatly underestimates the maximum leachate head over the liner for this specific landfill cell. Moore's 1980 method overestimates the maximum leachate head by 9%, and Moore's 1983 method underestimates the maximum leachate head by 26% in the above example. The result from Giroud's 1992 method is almost same as that from McEnroe's 1993 method. These two methods differ by only 0.2%.

In order to conduct further comparisons among these four methods for various design conditions of landfill cells, calculations were conducted for a landfill cell with different leachate drainage distances and slopes, various values of hydraulic conductivity of the drainage layer, and different inflow rates for all four methods. Fig. 3 shows the relationship between the maximum leachate head and the horizontal drainage distance for $r = 3$ mm/day, $k = 0.01$ cm/s, and $S = 2\%$. Fig. 4 shows the relationship between the maximum leachate head and the drainage slope for $r = 3$ mm/day, $k = 0.01$ cm/s, and $L = 25$ m. Fig. 5 shows the relationship between the maximum leachate head and the inflow rate for $L = 25$ m, $k = 0.01$ cm/s, and $S = 2\%$. Fig. 6 shows the relationship between the maximum leachate head and the hydraulic conductivity of the drainage layer for $r = 3$ mm/day, $L = 25$ m, and $S = 2\%$. Taken collectively, these four figures result in similar trends.

1. Moore's 1980 method always overpredicts McEnroe's 1993 method and in many cases by a considerable amount;
2. Moore's 1983 method always underpredicts McEnroe's 1993 method and in many cases by a considerable amount; and
3. Giroud's 1992 method is in close agreement with McEnroe's 1993 method and in most cases the curves almost overlap one another.

For further detail in describing the behavior of these curves, see Qian et al. (2001).

Landfill Cover System

Assume that the sand drainage system of a landfill cover has the following properties and characteristics: hydraulic conductivity of the sand drainage layer of 0.01 cm/s, slope of 25%, horizontal distance from the most upstream point to the toe drain of 100 m, and an inflow rate of 3 mm/day. The maximum liquid head over the liner calculated by these different methods is as follows:

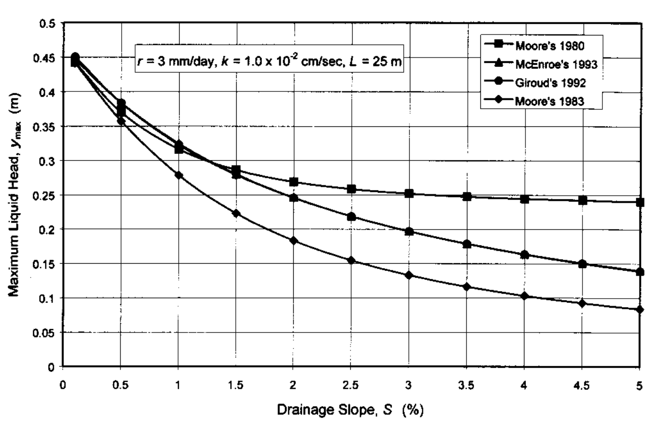

Fig. 4. Relationship between maximum liquid head and drainage slope for landfill cell from different calculation methods

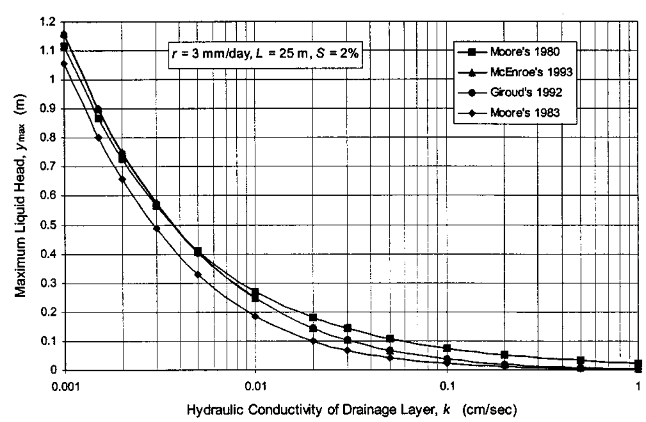

Fig. 6. Relationship between maximum liquid head and hydraulic conductivity of drainage layer for landfill cell from different calculation methods

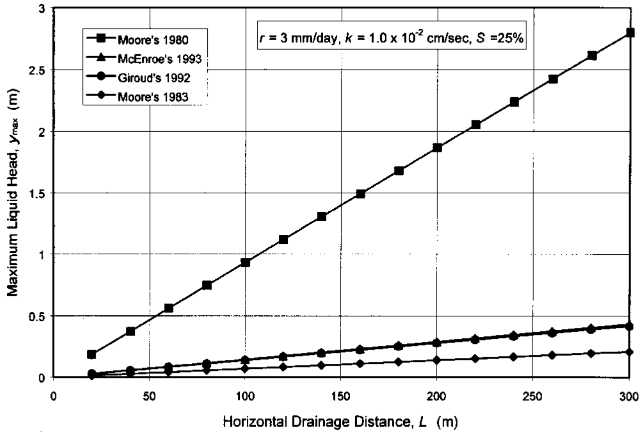

Fig. 7. Relationship between maximum liquid head and horizontal drainage distance for landfill cover from different calculation methods

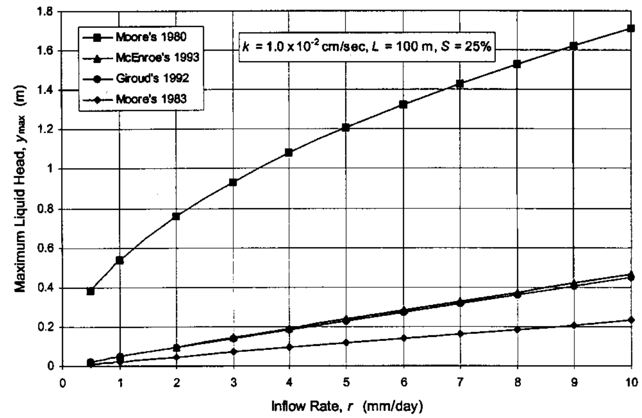

Fig. 9. Relationship between maximum liquid head and inflow rate for landfill cover from different calculation methods

Moore's 1980 method

$$y_{\max}=933 \text{ mm}$$

Moore's 1983 method

$$y_{\max}=69 \text{ mm}$$

Giroud's 1992 method

$$y_{\max}=139 \text{ mm}$$

McEnroe's 1993 method

$$y_{\max}=144 \text{ mm}$$

These values indicate that Moore's 1980 method greatly overestimates the maximum saturated depth in the final cover system compared to either McEnroe's 1993 or Giroud's 1992 method. Conversely, Moore's 1983 method greatly underestimates the maximum liquid head relative to these latter two methods. The result from Moore's 1980 method is almost 6.5 times that of McEnroe's 1993 method, whereas Moore's 1983 method yields a result less than 50% of that from either McEnroe's 1993 or Giroud's 1992 method. The depth calculated by these latter two methods differs by only 3.4%.

In order to conduct further comparisons for these four methods for various design conditions of landfill cover systems, calculations were also carried out using all four methods for a landfill cover with different drainage distances and slopes, various values of hydraulic conductivity of the drainage layer, and for different inflow rates. Fig. 7 shows the relationship between the maximum saturated depth and the horizontal drainage distance for $r=3$ mm/day, $k=0.01$ cm/s, and $S=25\%$. Fig. 8 shows the relationship between the maximum saturated depth and the drainage slope for $r=3$ mm/day, $k=0.01$ cm/s, and $L=100$ m. Fig. 9 shows the relationship between the maximum saturated depth and the inflow rate for $k=0.01$ cm/s, $L=100$ m, and $S=25\%$. Fig. 10 shows the relationship between the maximum leachate head and the hydraulic conductivity of the drainage layer for $r=3$ mm/day, $L=100$ m, and $S=2\%$.

A comparison of the results shown in Figs. 7–10 reveals that Moore's 1980 method always greatly overestimates McEnroe's 1993 method whereas Moore's 1983 method always underestimates the maximum saturated depth in the final cover system. The difference between the results of Giroud's 1992 method and McEnroe's 1993 method is less than 4% with a 25% drainage slope, about 5.4% with a $3(H){:}1(V)$ drainage slope, and up to 10.6% with a $2(H){:}1(V)$ drainage slope for various values of drainage distances, inflow rates, and hydraulic conductivity of the

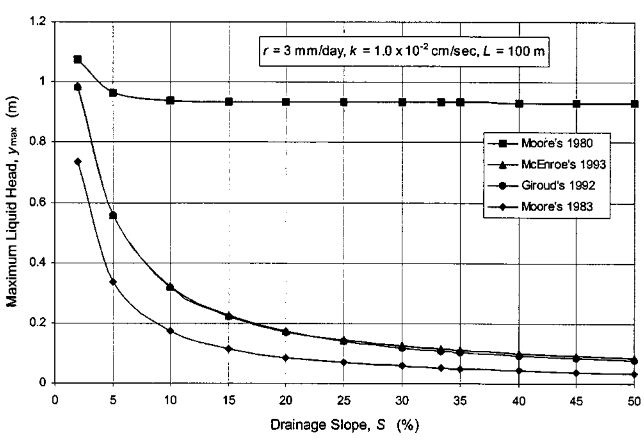

Fig. 8. Relationship between maximum liquid head and drainage slope for landfill cover from different calculation methods

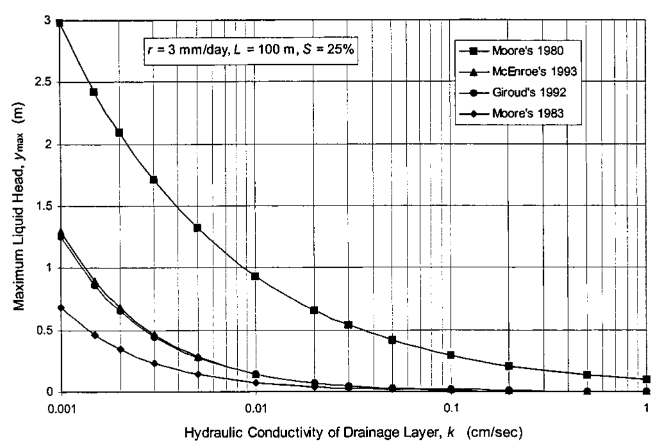

Fig. 10. Relationship between maximum liquid head and hydraulic conductivity of drainage layer for landfill cover from different calculation methods

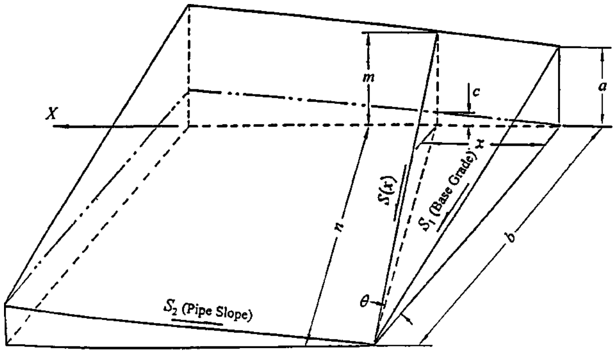

Fig. 11. Landfill cell floor grade

drainage layer. Additional commentary on the more precise differences in the four predictive methods can be found in Qian et al. (2001).

Effects of Base Grade and Pipe Slope on Maximum Leachate Head

Using the above methods, it can be seen that the factors affecting the maximum liquid head over the barrier include: (1) inflow rate, (2) hydraulic conductivity of the drainage layer, (3) horizontal drainage distance, and (4) slope of the drainage layer. If the values of the inflow rate and hydraulic conductivity of the drainage layer are kept constant, the maximum liquid head increases with increase of the horizontal drainage distance and decreases with increase of drainage slope.

Currently, when calculating the maximum leachate head for a landfill leachate collection system, many engineers assume that the slope of the drainage layer in the equations is equal to the cell base grade (e.g., 2%) and furthermore that the horizontal drainage distance is equal to the horizontal distance from the upstream boundary to the leachate collection pipe, which is perpendicular to the pipe.

Actually, the bottom floor of a landfill cell usually has a two-dimensional slope including a pipe slope (e.g., 1%) and a cell base slope that is perpendicular to the pipe (e.g., 2%). The typical landfill bottom floor is as shown in Fig. 11. Liquids always flow along the maximum grade. The maximum grade in Fig. 11 is not perpendicular to the leachate collection pipes. Thus, the maximum horizontal drainage distance from upstream boundary to the leachate collection pipe should be larger than the horizontal distance from upstream boundary to the pipe, which is perpendicular to the pipe (Qian 1994). The actual leachate flow grade S and the actual horizontal leachate flow distance from upstream boundary to the leachate collection pipe L can be calculated as follows.

Assume the slope of the bottom liner grade perpendicular to the leachate collection pipe (i.e., base grade) is S_1 and the slope of the leachate collection pipe is S_2

$$S_1 = a/b, \quad S_2 = c/x$$

$$m = a + c = S_1 b + S_2 x, \quad n = (b^2 + x^2)^{0.5}$$

In Fig. 11, it can be seen that the landfill floor slope varies with changes of x. The landfill floor slope $S(x)$ can be expressed as follows:

$$S(x) = \frac{m}{n} = \frac{S_1 b + S_2 x}{(b^2 + x^2)^{0.5}}$$

$$\frac{dS(x)}{dx} = -\frac{S_1 bx}{(b^2+x^2)^{1.5}} + \frac{S_2}{(b^2+x^2)^{0.5}} - \frac{S_2 x^2}{(b^2+x^2)^{1.5}}$$

$$= \frac{S_2 b^2 - S_1 bx}{(b^2+x^2)^{1.5}}$$

The landfill floor slope is a maximum when $dS(x)/dx = 0$.

$$\frac{S_2 b^2 - S_1 bx}{(b^2+x^2)^{1.5}} = 0$$

Since $b^2 + x^2$ can never be equal to 0

$$S_2 b^2 - S_1 bx = 0$$

$$x = (S_2/S_1) b$$

For the maximum floor slope, S

$$m = S_1 b + S_2 x = S_1 b + S_2 \cdot (S_2/S_1) \cdot b = \frac{(S_1^2 + S_2^2) \cdot b}{S_1}$$

$$n = (b^2+x^2)^{0.5} = [b^2 + (S_2/S_1)^2 b^2]^{0.5} = b \cdot [1 + (S_2/S_1)^2]^{0.5}$$

$$= \frac{(S_1^2+S_2^2)^{0.5} b}{S_1}$$

Thus

$$S = \frac{m}{n} = \frac{(S_1^2+S_2^2) \cdot b}{(S_1^2+S_2^2)^{0.5} \cdot b}$$

$$S = (S_1^2 + S_2^2)^{0.5} \tag{11}$$

where S = maximum landfill floor slope (i.e., actual leachate drainage slope); S_1 = base grade; and S_2 = pipe slope. The horizontal distance from upstream boundary to the leachate collection pipe along the actual leachate drainage slope is

$$L = b \cdot [1 + (S_2/S_1)^2]^{0.5} \tag{12}$$

where L = horizontal distance from upstream boundary to the leachate collection pipe along the actual leachate drainage slope (m); and b = horizontal distance from upstream boundary to the leachate collection pipe, which is perpendicular to the leachate collection pipe (m). The angle between L and b can be calculated from the following equation:

$$\theta = \cos^{-1}(b/L) \tag{13}$$

where θ = angle between L and b (deg).

As an illustration of the above concept, consider a landfill cell, $r = 3$ mm/day, $k = 1.0 \times 10^{-2}$ cm/s, $b = 30$ m, the maximum leachate head calculated with McEnroe's 1993 method for eight different combinations of base grade S_1 and pipe slope S_2 are listed in Table 1. For this landfill cell, if the pipe slope is not considered (i.e., Case 1 in Table 1) the calculated maximum leachate head is 295 mm, which meets the regulatory requirement. If a pipe slope of 1% is considered (i.e., Case 2), the calculated maximum leachate head becomes 312 mm, which does not meet the regulatory requirement. But this latter condition is characteristic of the actual field conditions. Thus, both base grade and pipe slope must be considered in the calculation of the maximum leachate head for design of a leachate collection system.

By comparing results from Cases 2 to 5 shown in Table 1, Case 2 has the lowest drainage slope (i.e., $S = 2.24\%$) and Case 5 has the highest drainage slope (i.e., $S = 10.20\%$). Note that Case 2 has the lowest maximum leachate head (i.e., $y_{\max} = 312$ mm), whereas Case 5 has the highest maximum leachate head (i.e.,

Table 1. Maximum Leachate Head for Different Combinations of Base Grade and Pipe Slope

Case	S_1	S_2	S	θ (deg)	L (m)	γ_{max} (mm)
1	0.02	0	0.02	0	30	295
2	0.02	0.01	0.0224	26.6	33.5	312
3	0.02	0.02	0.0283	45.0	42.4	346
4	0.02	0.05	0.0539	68.2	80.8	427
5	0.02	0.10	0.1020	78.7	153.0	482
6	0.01	0.005	0.0112	26.6	33.5	420
7	0.01	0.01	0.0141	45.0	42.4	487
8	0.01	0.02	0.0224	63.4	67.0	624

Note: $r=3$ mm/day; $k=1.0\times10^{-2}$ cm/s; and $b=30$ m.

$y_{max}=482$ mm) because the actual horizontal drainage distance for Case 2 (i.e., $L=33.5$ m) is much shorter than that for Case 5 (i.e., $L=153.0$ m).

By comparing Case 2 to Case 8, these two cases have the same drainage slope (i.e., $S=2.24\%$), yet the maximum leachate head for Case 8 (i.e., $y_{max}=624$ mm) is much higher than that for Case 2 (i.e., $y_{max}=312$ mm) because the combination of the base grade and pipe slope for Case 8 causes a longer actual horizontal drainage distance (i.e., $L=67.0$ m) than that for Case 2 (i.e., $L=33.5$ m) (Fig. 12).

In addition, by comparing Case 6 to Case 8, it can be seen that both drainage slope and horizontal drainage distance for Case 6 (i.e., $S=1.12\%$ and $L=33.5$ m) are half of that for Case 8 (i.e., $S=2.24\%$ and $L=67.0$ m) but the maximum leachate head for Case 6 (i.e., $y_{max}=420$ mm) is much lower than that for Case 8 (i.e., $y_{max}=624$ mm). This illustrates that a reduced drainage distance is more effective than an increase in drainage slope to lower the maximum liquid head over the barrier. This finding can be seen more clearly from the curves of McEnroe's 1993 method in Figs. 3, 4, 7, and 8. Figs. 3 and 7 show that there is a linear relationship between the horizontal drainage distance and maximum liquid head. When the horizontal drainage distance increases five times, the maximum liquid head also increases five times (e.g., in Fig. 3, $L=10$ m, $y_{max}=98$ mm; $L=50$ m, $y_{max}=492$ mm; and in Fig. 7, $L=40$ m, $y_{max}=58$ mm; $L=200$ m, $y_{max}=288$ mm). It can be observed from Figs. 4 and 8 that there is a nonlinear relationship between the drainage slope and maximum liquid head. When the drainage slope increases five times, the decrease of maximum liquid head is less than five times. For example, from Fig. 4, when the drainage slope increases from 0.6 to 3%, i.e., the increase expressed as a ratio is 5, and the maximum liquid head decreases from 0.370 to 0.197 m, i.e., the decrease in the maximum liquid head expressed as a ratio is only 1.9. From Fig. 8, when the drainage slope increases from 5 to 25%, i.e., the increase ratio of the drainage slope is also 5, whereas the maximum liquid head decreases from 559 to 144 mm, i.e., the decrease ratio of the maximum liquid head is 3.9.

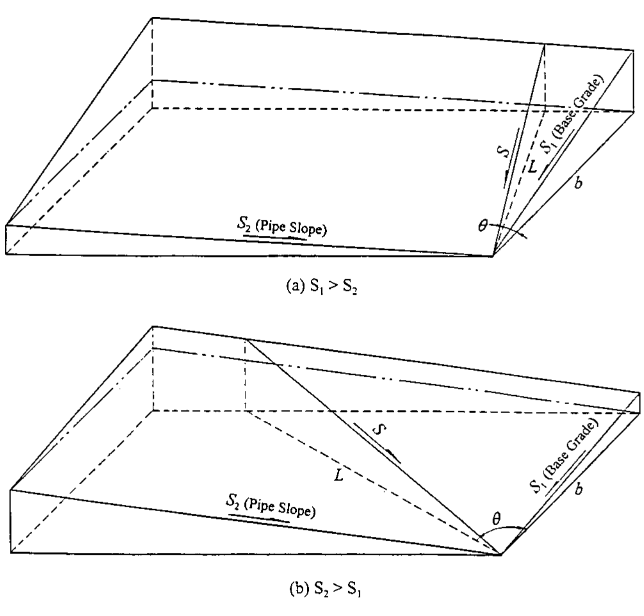

Fig. 12. Actual leachate flow grade and horizontal drainage distance for different combinations of base grade and pipe slope: (a) $S_1>S_2$ and (b) $S_1<S_2$

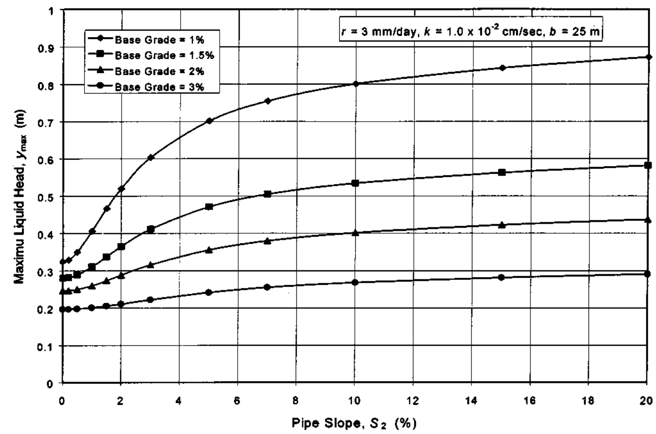

Fig. 13. Effect of pipe slope on maximum liquid head with various landfill base grades

For canyon-fill landfills (also called valley-fill landfills), there is usually a relatively steep natural subbase slope along the valley direction. Thus, the pipe slope may be much steeper than 1 or 2%, typical of values used in flat areas. Fig. 13 shows the effect of pipe slope on the maximum liquid head calculated with McEnroe's 1993 method with various landfill base grades, and Fig. 14 shows the effect of pipe slope on the actual horizontal drainage distance calculated with Eq. (12) with various landfill base grades for a landfill cell with $r=3$ mm/day, $k=1.0\times10^{-2}$ cm/s, $b=25$ m. The results in Fig. 13 show that the maximum leachate head increases with increase of pipe slope. The increasing trend

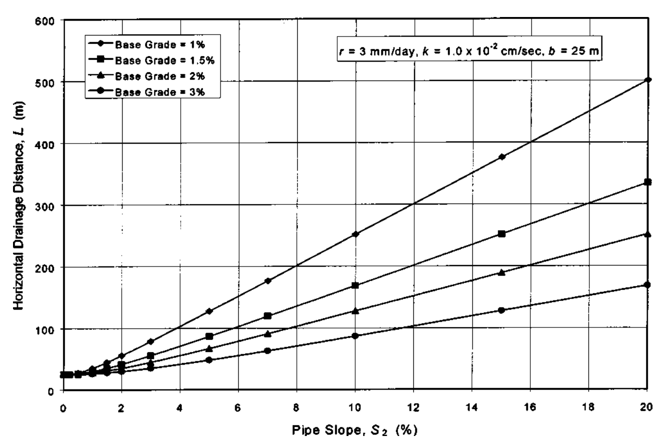

Fig. 14. Effect of pipe slope on horizontal drainage distance with various landfill base grades

of the maximum leachate head with increase of the pipe slope becomes less with a concomitant increase of base grade. Fig. 13 also shows that if the pipe slope is greater than 3%, a 2% based grade cannot keep the maximum leachate head less than 300 mm. On the other hand, if a cell subbase is graded to a 3% slope, the maximum leachate head can be maintained less than 300 mm even if the pipe slope is increased to 20%. Fig. 14 shows that the horizontal drainage distance increases with an increase of pipe slope. The increasing trend of the horizontal drainage distance with increase of the pipe slope also becomes less with a concomitant increase in base grade.

The results of the eight cases listed in Table 1 and plotted in Figs. 13 and 14 can be summarized as follows:

1. Base slope is not the only critical parameter affecting leachate head on the liner when values of *r*, *k*, and *b* are fixed. A simple inverse linear relationship does not exist between leachate head on a liner and its slope. This is seen by comparing Cases 2, 3, 4, and 5, and between Cases 6 and 8, respectively.
2. Different combinations of base slope and pipe slope can directly affect the actual drainage distance as can be seen by comparing Cases 2 with 8.
3. A change in drainage distance has a greater effect on the maximum leachate head than a change of drainage slope.

Equivalent Hydraulic Conductivity for Unconfined Seepage in Multilayered Media

If the liquid head over a barrier in landfill cover system is greater than the thickness of the drainage layer, a part of the protective layer will change its original function from a filtration layer to a drainage layer. The hydraulic conductivity of the protective layer is different than the hydraulic conductivity of the drainage layer. It is usually much lower than that of the drainage layer. This situation usually occurs in the final cover system during and after a heavy storm. Whether the maximum liquid head can be properly estimated in the cover is critical for evaluating the long-term stability of landfill cover system. This same situation can occur in leachate collection system when using geosynthetic drainage materials supplemented by overlying natural soil layers, i.e., sands or gravels.

Currently, two methods are used to treat this scenario. One method is to simply use the calculated maximum liquid head from the equations without considering the thickness of drainage layer. If the calculated maximum liquid head is greater than the thickness of the drainage layer, another approach is used. In this case the following equation is used to calculate the average hydraulic conductivity of both drainage layer and a layer overlying the drainage layer:

$$k_{\text{avg}}=\frac{k_1T_1+k_2T_2}{(T_1+T_2)} \tag{14}$$

where k_{avg} = average hydraulic conductivity (cm/s); k_1 = hydraulic conductivity of drainage layer (cm/s); k_2 = hydraulic conductivity of a layer overlying the drainage layer (cm/s); T_1 = thickness of drainage layer (cm); and T_2 = thickness of a layer overlying the drainage layer. Then, the calculated average hydraulic conductivity is used to recalculate the maximum liquid head. Unfortunately, this method is only suitable for confined seepage condition, i.e., both of the layers are fully saturated. For the present condition, it is in an unconfined seepage condition, i.e., only a part of the layer overlying the drainage layer is saturated, but the actual saturated depth in this layer is not known.

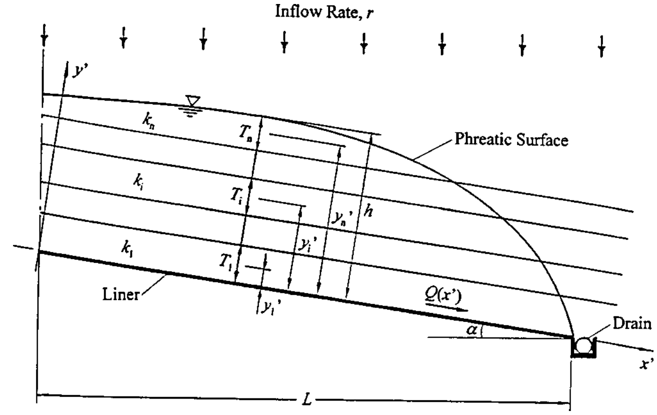

Fig. 15. Phreatic surface in multilayered drainage media

From Figs. 6 and 10, it can be seen that the maximum liquid head is very sensitive to changes of hydraulic conductivity. When reducing hydraulic conductivity from 0.01 to 0.001 cm/s, the maximum liquid head increase from 0.260 to 1.264 m in Fig. 6 and from 0.144 to 1.299 m in Fig. 10.

One may suppose that the first of the above mentioned methods may underestimate the maximum liquid head. When the calculated maximum liquid head is greater than the thickness of the drainage layer it means that the liquid head intrudes into the overlying protective layer, or solid waste layer. These materials usually have a much lower hydraulic conductivity. This means that the actual maximum liquid head will be greater than the calculated value. On the other hand, the second method may overestimate the maximum liquid head because this method is only suitable for the confined seepage condition, i.e., both two layers are fully saturated. For the current or unconfined seepage condition, only a part of the layer over the drainage layer is saturated. Therefore, the average hydraulic conductivity calculated from Eq. (14) must be lower than the actual equivalent hydraulic conductivity for the total saturated depth. This makes the calculated maximum liquid head greater than the actual value.

Because the actual saturated depth in the layer overlying the drainage layer is not known, the key question becomes how to determine the saturated depth in the multilayered drainage media and how to calculate the equivalent hydraulic conductivity for the saturated layered drainage media. Then, the actual maximum liquid head can be calculated from either the McEnroe's 1993 or Giroud's 1992 method.

If the hydraulic conductivity of the drainage layer varies with saturated depth, the Girinskii's potential, Φ, in an unconfined seepage condition can be expressed as the following equation (Girinskii 1946; Bear 1972):

$$\Phi=\int_0^h (h-y')k(y')dy' \tag{15}$$

See Fig. 15 for definitions of the various terms in the above equation.

In the above expression, *h* is liquid depth at any location, which is perpendicular to the flow direction. The differentiation of Eq. (15) along the flow direction is equal to the amount of the seepage flow

$$Q(x')=\partial\Phi/\partial x'=(\partial h/\partial x')\int_0^h k(y')dy' \tag{16}$$

When the hydraulic conductivity is a constant, Eq. (15) becomes

$$\Phi = \int_0^h (h-y')\cdot k dy' = kh\cdot(h-h/2) = 0.5kh^2 \tag{17}$$

Fig. 15 shows a cross section of multilayered drainage media. The thickness of each layer, from bottom to top, is $T_1, T_2, \ldots, T_i, \ldots, T_{n-1}, T_n$, respectively, and the hydraulic conductivity of each layer is $k_1, k_2, \ldots, k_i, \ldots, k_{n-1}, k_n$, respectively. The distance from the barrier to the center of each saturated layer is $y_1', y_2', \ldots y_i', \ldots y_{n-1}', y_n'$, respectively.

$$\Phi = k_1T_1\cdot(h-y_1') + k_2T_2\cdot(h-y_2') + \cdots + k_iT_i\cdot(h-y_i') + \cdots + k_{n-1}T_{n-1}\cdot(h-y_{n-1}') + k_nT_n\cdot(h-y_n') \tag{18}$$

From Fig. 15

$$y_1' = T_1/2$$

$$\vdots$$

$$y_i' = T_1 + T_2 + \cdots + T_i/2$$

$$\vdots$$

$$y_n' = T_1 + T_2 + \cdots + T_i + \cdots + T_{n-1} + T_n/2$$

Therefore

$$\Phi = k_1T_1(h-T_1/2) + \cdots + k_iT_i[h-(T_1+T_2+\cdots+T_i/2)] + \cdots + k_nT_n\cdot[h-(T_1+T_2+\cdots+T_i+\cdots+T_{n-1}+T_n/2)] \tag{19}$$

Assume k_{eq} is equivalent hydraulic conductivity under the phreatic surface

$$\Phi = k_{eq}h\cdot(h-h/2) = 0.5k_{eq}h^2 \tag{20}$$

Since Eq. (20) = Eq. (19)

$$0.5k_{eq}h^2 = k_1T_1\cdot(h-T_1/2) + \cdots + k_iT_i\cdot[h-(T_1+T_2+\cdots+T_i/2)] + \cdots + k_nT_n\cdot[h-(T_1+T_2+\cdots+T_{n-1}+T_n/2)] \tag{21}$$

For multilayered drainage media, the equivalent hydraulic conductivity under the phreatic surface can be calculated from the following equation:

$$k_{eq} = 2\cdot\{k_1T_1\cdot(h-T_1/2) + \cdots + k_iT_i\cdot[h-(T_1+T_2+\cdots+T_i/2)] + \cdots + k_nT_n\cdot[h-(T_1+T_2+\cdots+T_{n-1}+T_n/2)]\}/h^2 \tag{22}$$

For a two-layered drainage media that is typical for a landfill drainage layer or final cover system, Eq. (21) becomes

$$0.5k_{eq}h^2 = k_1T_1\cdot(h-T_1/2) + k_2T_2[h-(T_1+T_2/2)]$$

Because $h = T_1 + T_2$

$$0.5k_{eq}\cdot(T_1+T_2)^2 = k_1T_1\cdot(T_1+T_2-T_1/2) + k_2T_2[T_1+T_2-T_1-T_2/2)]$$

$$k_{eq}(T_1+T_2)^2 = k_1(T_1+T_2)^2 + (k_2-k_1)T_2^2$$

Therefore, for two-layered drainage media (Fig. 16), the equivalent hydraulic conductivity under the phreatic surface can be calculated from the following equation:

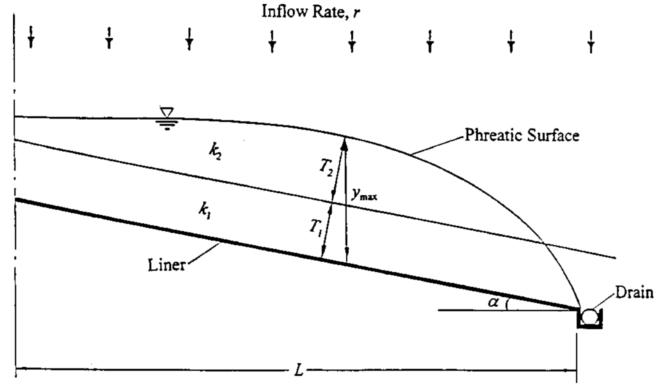

Fig. 16. Phreatic surface in two-layered drainage media, typical of leachate collection and cover soil drainage situations

$$k_{eq} = k_1 + (k_2-k_1)\frac{T_2^2}{(T_1+T_2)^2} \tag{23a}$$

For a two-layered drainage media, if T_1 and T_2 are at the location of the maximum saturated depth, the maximum liquid head y_{max} can be calculated as follows:

$$y_{max} = (T_1+T_2)/\cos\alpha \tag{24a}$$

Because the value of T_2 is unknown, a trial and error method can be used to calculate the maximum leachate head in the two-layered drainage media.

Calculation of Maximum Liquid Head in Multilayered Media

For two-layered drainage media, the maximum liquid head over the barrier can be calculated from the following steps:

1. Only consider the first layer T_1 as a leachate drainage layer. Use k_1 and McEnroe's 1993 method or Giroud's 1992 method [for a drainage slope less than $3(H):1(V)$] to calculate the maximum liquid head $(y_{max})_{calculated}$.
2. If $(y_{max})_{calculated} \leq T_1/\cos\alpha$, the calculation has been completed. If $(y_{max})_{calculated} > T_1/\cos\alpha$, it means the layer overlying the drainage layer must be considered as an additional liquid drainage layer to recalculate the actual maximum liquid head in the double-layered drainage media.
3. Assume a T_2 value.
4. Use Eq. (23) to calculate the equivalent hydraulic conductivity k_{eq} according to assumed T_2.
5. Use Eq. (24) to calculate the assumed maximum liquid head $(y_{max})_{assumed}$ according to the assumed T_2.
6. Use the equivalent hydraulic conductivity k_{eq} from step (4) and McEnroe's 1993 or Giroud's 1992 method to calculate the maximum liquid head $(y_{max})_{calculated}$.
7. Compare the calculated maximum liquid head $(y_{max})_{calculated}$ from step (6) with the assumed maximum liquid head $(y_{max})_{assumed}$ from step (5).
8. If $(y_{max})_{calculated} \neq (y_{max})_{assumed}$, assume another T_2 and repeat steps (3)–(8) until $(y_{max})_{calculated} = (y_{max})_{assumed}$.

For the multilayered drainage media, assuming a T_n in a proper layer and using the similar method as above and Eqs. (22) and (25), the maximum liquid head also can be calculated by a trial and error method.

Consider a final cover for a municipal solid waste landfill that is constructed on a slope of 25% with a horizontal distance of 100

m. A sand drainage layer with a hydraulic conductivity of 0.01 cm/s is 0.3 m thick over a geomembrane and compacted soil composite barrier. A 0.6 m silty sandy is placed over the drainage layer as a protective layer with a hydraulic conductivity of 0.001 cm/s. The amount of rainfall percolating though the cover is estimated to be 10 mm/day. The estimated maximum saturated depth within the landfill cover system can be calculated as follows:

$$k_1 = 0.01 \text{ cm/s}, \quad k_2 = 0.001 \text{ cm/s}$$

$$T_1 = 0.3 \text{ m}, \quad L = 100 \text{ m}$$

$$r = 10 \text{ mm/day}, \quad S = \tan\alpha = 0.25,$$

$$\alpha = 14.04°$$

1. Only consider 0.3 m sand as a drainage layer to calculate maximum saturated depth and use McEnroe's 1993 or Giroud's 1992 method to calculate the maximum liquid head

   $$(y_{\max})_{\text{calculated}} = 0.463 \text{ m}$$

   Because $(y_{\max})_{\text{calculated}} = 0.463 \text{ m} > T_1/\cos\alpha = 0.309 \text{ m}$, the silty sand protective layer must be considered as a drainage layer to calculate the maximum saturated depth (Fig. 16).
2. Assume $T_2 = 0.23$ m
   Calculate the equivalent hydraulic conductivity

   $$k_{\text{eq}} = k_1 + (k_2 - k_1)\frac{T_2^2}{(T_1+T_2)^2}$$

   $$= 0.01 + (0.001 - 0.01)\frac{23^2}{(30+23)^2} = 0.00831 \text{ cm/s} \quad (23b)$$

   Calculate the assumed maximum liquid head

   $$(y_{\max})_{\text{assumed}} = (T_1+T_2)/\cos\alpha = (0.3+0.23)/\cos 14.04°$$

   $$= 0.546 \text{ m} \quad (24b)$$

   Use $k_{\text{eq}} = 0.0831$ cm/s and McEnroe's 1993 or Giroud's 1992 method to calculate the maximum liquid head

   $$(y_{\max})_{\text{calculated}} = 0.553 \text{ m}$$

   Because $(y_{\max})_{\text{assumed}} = 0.546 \text{ m} < (y_{\max})_{\text{calculated}} = 0.553 \text{ m}$, the actual T_2 must be greater than 0.23 m.
3. Assume $T_2 = 0.25$ m
   Calculate the equivalent hydraulic conductivity

   $$k_{\text{eq}} = 0.01 + (0.001 - 0.01)\frac{25^2}{(30+25)^2} = 0.00814 \text{ cm/s}$$

   Calculate the assumed maximum liquid head

   $$(y_{\max})_{\text{assumed}} = (0.3+0.25)/\cos 14.04° = 0.567 \text{ m}$$

   Use $k_{\text{eq}} = 0.0814$ cm/s and McEnroe's 1993 or Giroud et al.'s 1992 method to calculate the maximum liquid head.

   $$(y_{\max})_{\text{calculated}} = 0.564 \text{ m}$$

   Because $(y_{\max})_{\text{assumed}} = 0.567 \text{ m} > (y_{\max})_{\text{calculated}} = 0.564 \text{ m}$, the actual T_2 is between 0.23 and 0.25 m
4. Assume $T_2 = 0.243$ m
   Calculate the equivalent hydraulic conductivity

   $$k_{\text{eq}} = 0.01 + (0.001 - 0.01)\cdot\frac{24.3^2}{(30+24.3)^2} = 0.0820 \text{ cm/s}$$

   Calculate the assumed maximum liquid head

   $$(y_{\max})_{\text{assumed}} = (0.3+0.243)/\cos 14.04° = 0.560 \text{ m}$$

   Use $k_{\text{eq}} = 0.0820$ cm/s and McEnroe's 1993 or Giroud's 1992 method to calculate the maximum saturated depth

   $$(y_{\max})_{\text{calculated}} = 0.560 \text{ m}$$

Because $(y_{\max})_{\text{assumed}} = 0.560 \text{ m} = (y_{\max})_{\text{calculated}} = 0.560$ m, the actual maximum saturated depth in the final cover system is 0.560 m.

If the equivalent hydraulic conductivity for the saturated double-layered drainage media is not considered

$$y_{\max} = 0.457 \text{ m (underestimated)}$$

If Eq. (14) is used to determine average hydraulic conductivity for both layers (i.e., $k_{\text{avg}} = 0.004$ cm/s)

$$y_{\max} = 1.084 \text{ m (overestimated)}$$

The actual maximum liquid head can then be used as a seepage head to conduct a final cover stability analysis (see Soong and Koerner 1996; Koerner and Daniel 1997; Qian et al. 2001).

When a geonet or geocomposite is used as a drainage layer, a hydraulic transmissivity is generated from the laboratory test instead of hydraulic conductivity. This is related to

$$\Theta = kt \quad (25)$$

where Θ = hydraulic transmissivity (cm^2/s); k = hydraulic conductivity of the geosynthetic (cm/s); and t = thickness of the transmissive component of the geonet or geocomposite (cm).

Hydraulic conductivity of geonet or geocomposite can be calculated from the above formula. Geonets or geocomposites can handle significantly larger flow rates compared to soil, however the flow within their apertures is not laminar flow; it is probably turbulent flow. The transmissivity values of geonets and geocomposites change with changes of hydraulic gradient and overburden pressure. Thus, caution on using the hydraulic transmissivity of the geosynthetic must be expressed.

To ensure long-term performance, the hydraulic design of liquid drainage layers must ensure that the liquid drainage layers have sufficient flow capacity under the conditions that exist in the field during the entire design life of the liquid drainage layers. The flow capacity of a liquid drainage layer in the field can be reduced by a variety of mechanisms that depend on the applied load, time, contact with adjacent materials, and environmental conditions (e.g., presence of chemicals, biological activity, and temperature) for the geosynthetic drainage layer and the potential clogging (e.g., particulate, chemical, and biological clogging) for the granular drainage layer (Koerner and Koerner 1995; Rowe 1998; Fleming et al. 1999; Giroud et al. 2000). The detailed descriptions for determining the long-term hydraulic transmissivity of the geosynthetic drainage layer and the long-term hydraulic conductivity of the granular drainage layer can be found in Koerner (1998), Giroud et al. (2000), and Qian et al. (2001).

Conclusions

The following conclusions can be drawn from the results of this study of maximum leachate head estimates over a landfill liner and/or the maximum saturated depth in a final cover system:

1. Comparisons of four current available methods for calculating the maximum liquid head over landfill barriers, indicate

that McEnroe's 1993 method is suitable for the general analysis and design of drainage system for landfill covers and bottom liners. Giroud's 1992 method is recommended for analysis and design of drainage systems for landfill covers and bottom liners with a drainage slope less than $3(H):1(V)$. It must be noted that the methods described herein are only reliable for a ''free drainage condition.'' This condition implies the liquid level in the drainage trench is always below the upper edge of the trench, and has no effect on the saturated-depth profile over the liner.

2. Pipe slope is a very important parameter affecting the maximum leachate head on the liner. Different combinations of base grade and pipe slope can directly affect the actual drainage distance. If the pipe slope is steeper than base slope, this condition will result in a longer actual drainage distance which in turn causes a higher leachate head on the liner.
3. A change in drainage distance has more effect on the maximum leachate head than a change of drainage slope.
4. A method for calculating the equivalent hydraulic conductivity of multilayered drainage media in an unconfined seepage condition is developed based on the Girinskii's potential theory in this paper.
5. A method for calculating the maximum liquid head in multilayered drainage media is also developed in this paper. The key for calculating the maximum liquid head in multilayered drainage media is to determine the equivalent hydraulic conductivity of the drainage media under the phreatic surface.
6. Whether the maximum liquid head in the landfill cover can be properly estimated in the worst condition is critical for evaluating the long-term stability of landfill cover system.
7. The factors affecting the maximum liquid head on the liner or cover barrier include inflow rate, hydraulic conductivity of the drainage layer (or equivalent hydraulic conductivity under the phreatic surface for multilayered drainage media), actual horizontal drainage distance from upstream boundary to downstream outlet, and maximum slope of the drainage layer.

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